Time-Varying Optimization and Real-Time Optimal Power Flow

Award #1739355
Caltech PI: Steven Low, slow@caltech.edu

Challenges

Finding exact optimal operations may not be appropriate in ***time-varying*** setting:

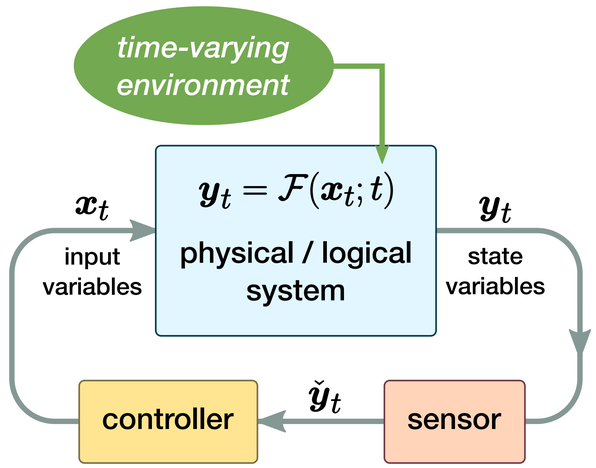
- It can be slow for large systems.
- The system / environment may have changed a lot after an exact solution is found.

What we need

Online, real-time approach

Feedback-based

Ability to ***track*** the optimal solutions



Goal:

$$x_t^* = \arg\min_{x_t} \; c_t(x_t)$$
$$\text{s.t.} \; h_t(y_t) \le 0$$
$$x_t \in \mathcal{X}_t$$

time-varying cost function

state variable constraints

input variable constraints

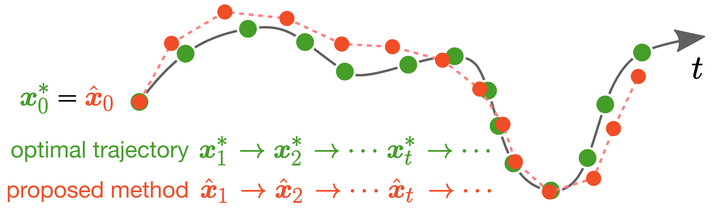
Solution

Feedback-based Primal-Dual Gradient Algorithm

$$\hat{x}_t = \mathcal{P}_{\mathcal{X}_t}\left[\hat{x}_{t-1} - \alpha\left(\nabla c_t(\hat{x}_{t-1}) + (H_t(\check{y}_t)J_t(\hat{x}_{t-1}, \check{y}_t))^T \hat{\lambda}_{t-1}\right)\right]$$
$$\hat{\lambda}_t = \mathcal{P}_{\mathbb{R}^+_m}\left[\hat{\lambda}_{t-1} + \beta\left(h_t(\check{y}_t) - \epsilon\hat{\lambda}_{t-1}\right)\right]$$

Feedback-based Second-Order Penalty Algorithm

$$\hat{x}_t = \mathcal{P}^{B_t}_{\mathcal{X}_t}\left[\hat{x}_{t-1} - B_t^{-1}\nabla f_t(\hat{x}_{t-1})\right]$$



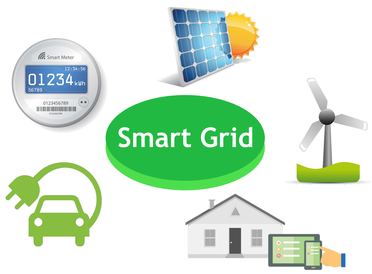
Theorem: $\|\hat{x}_t - x_t^*\| \le C_1\left(\sigma + C_2 \sup_t \|\lambda_t^*\| + C_3\Delta\right) \qquad \sigma := \sup_t \|x_t^* - x_{t-1}^*\|$

✓ Efficiency ✓ Robustness to Errors

✓ Guaranteed Tracking Performance

Applications

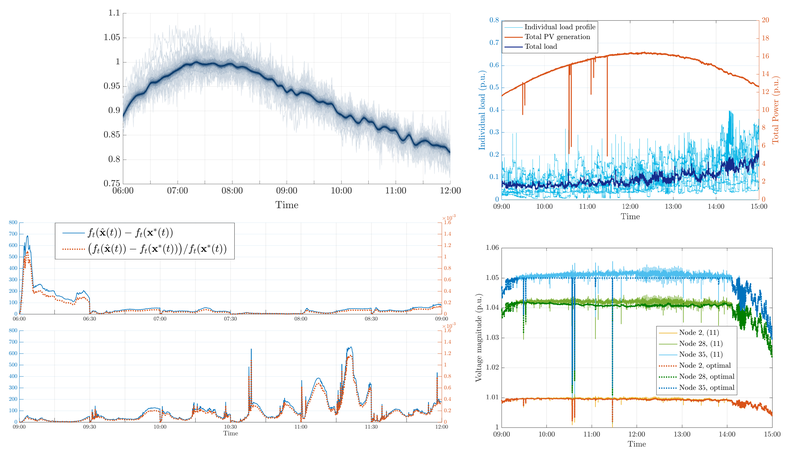
Real-Time Optimal Power Flow



Fluctuations Intermittency

Fast Control Capabilities

Real-time Measurements



Related Publications

[1] Y. Tang K. Dvijotham and S. Low. "Real-time optimal power flow," IEEE Transactions on Smart Grid, 10.1109/TSG.2017.2704922

[2] Y. Tang and S. Low. "Distributed algorithm for time-varying optimal power flow," 56th IEEE Conference on Decision and Control, 10.1109/CDC.2017.8264138

[3] Y. Tang, E. Dall'anese, A. Bernstein, and S. Low. "A feedback-based regularized primal-dual gradient method for time-varying nonconvex optimization", to appear in the 57th IEEE Conference on Decision and Control, 2018.