Optimal Placement of Energy Storage in Distribution Networks

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Motivation

Energy storage devices

- shift generation and consumption across time
- help integrate renewable energy resources

Challenges

- Joint optimization over both ***time*** and ***space***
- Numerical algorithms available but few ***structural results***

Formulation

Total storage budget: $\sum_x B(x) \le B_{\text{tot}}$

At location x:

storage state of charge

$b(x,t) \in [0, B(x)]$

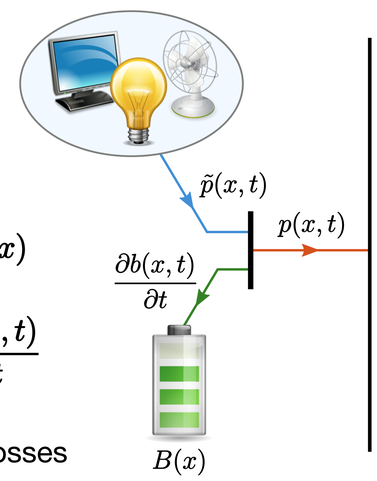
background injection

$\tilde{p}(x,t) \approx \alpha(x)p(t) + \beta(x)$

net power injection

$$p(x,t) = \tilde{p}(x,t) - \frac{\partial b(x,t)}{\partial t}$$

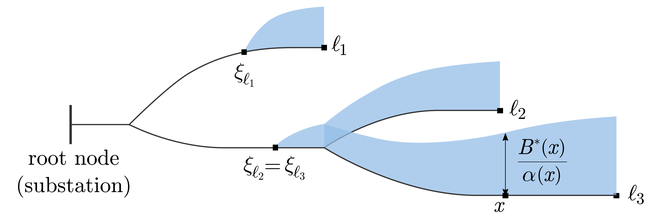
Goal: minimize network losses



Structure of Optimal Placement

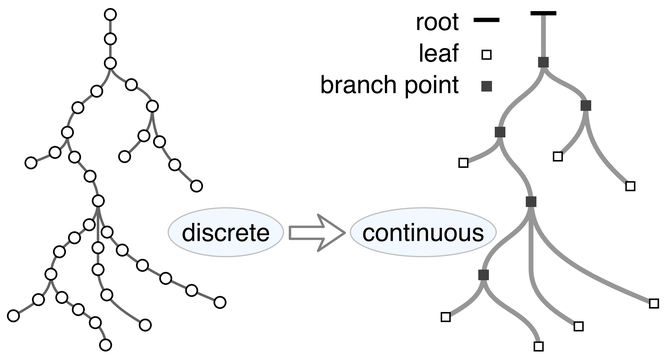
1. On each path connecting the root and a leaf, a point separates the path into two segments:
   - no storage on the segment containing the root
   - storage everywhere on the segment containing the leaf

2. The scaled optimal capacity $B^*(x)/\alpha(x)$ is ***increasing*** from the root to any leaf.



3. The locational marginal value of storage under optimal placement
   - is ***increasing*** from the root to any leaf,
   - is ***equalized*** over places where nonzero storage is allocated.

Solution



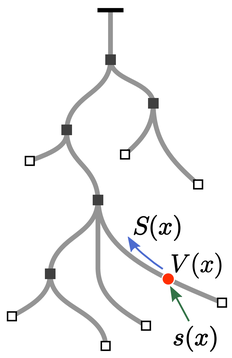
Power Flows on ***Continuous Trees***

$$S(x) = \int_{y \ge x} \left(s(y) - z(y)\frac{|S(y)|^2}{|V(y)|^2} \right) dy$$

$$|V(x)|^2 = |V_{\text{root}}|^2 + 2\int_{\text{root}}^{x} \text{Re}\left[z^*(y)S(y)\right]\, dy$$

✓ Continuous version of ***DistFlow Equations***

✓ Allow doing ***calculus*** on the network



Reference

[1] Y. Tang and S. H. Low. "Optimal placement of energy storage in distribution networks," IEEE Transactions on Smart Grid, 10.1109/TSG.2017.2711921

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