A Feedback-Based Regularized Primal-Dual Gradient Method for Time-Varying Optimization

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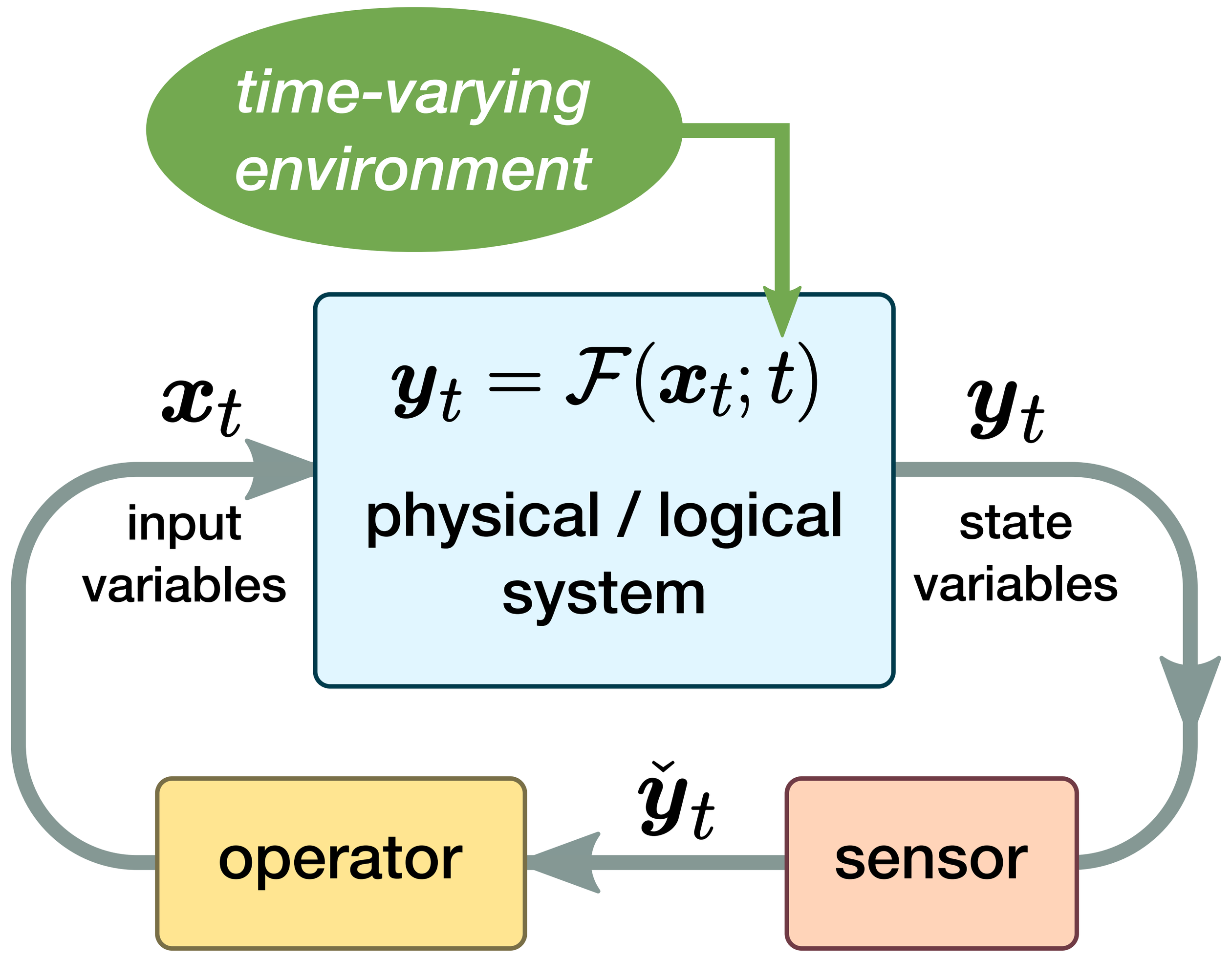
Introduction



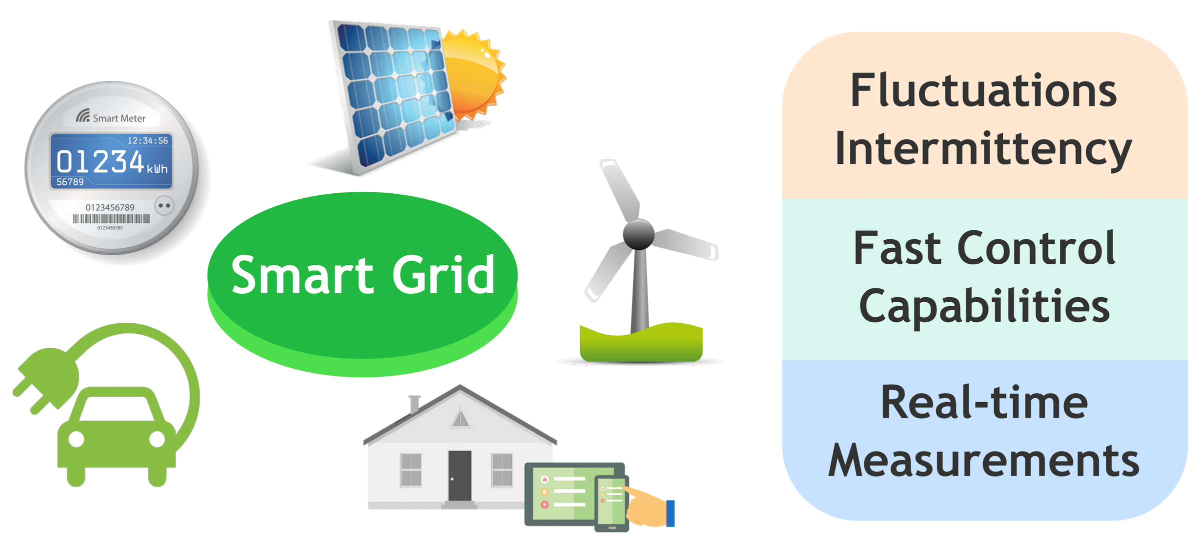
The operator's goal:

$$x_t^* = \arg\min_{x_t} \; c_t(x_t) \quad \text{(time-varying cost function)}$$
$$\text{s.t.} \; h_t(y_t) \le 0 \quad \text{(state variable constraints)}$$
$$x_t \in \mathcal{X}_t \quad \text{(input variable constraints (convex))}$$

This work: *online feedback-based* algorithms that *track* the optimal solutions x_t^*

Finding exact x_t^* may not be appropriate in *time-varying* setting:

- Can be slow for large systems
- The system / environment may have changed a lot after an exact solution has been found.



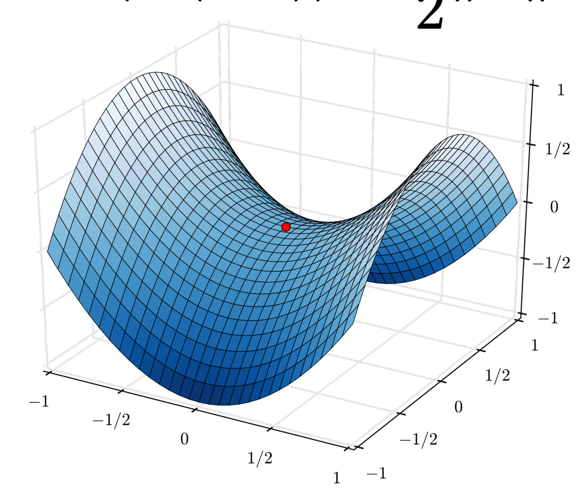
A typical example

Algorithm

Regularized Lagrangian: $\mathcal{L}_t(x,\lambda) = c_t(x) + \lambda^T h_t(\mathcal{F}(x;t)) - \frac{\epsilon}{2}\|\lambda\|^2$

Primal-dual gradient method:

$$\hat{x}_t = \mathcal{P}_{\mathcal{X}_t}\left[\hat{x}_{t-1} - \alpha\nabla_x \mathcal{L}_t(\hat{x}_{t-1}, \hat{\lambda}_{t-1})\right]$$
$$\hat{\lambda}_t = \mathcal{P}_{\mathbb{R}^m_+}\left[\hat{\lambda}_{t-1} + \beta\nabla_\lambda \mathcal{L}_t(\hat{x}_{t-1}, \hat{\lambda}_{t-1})\right]$$



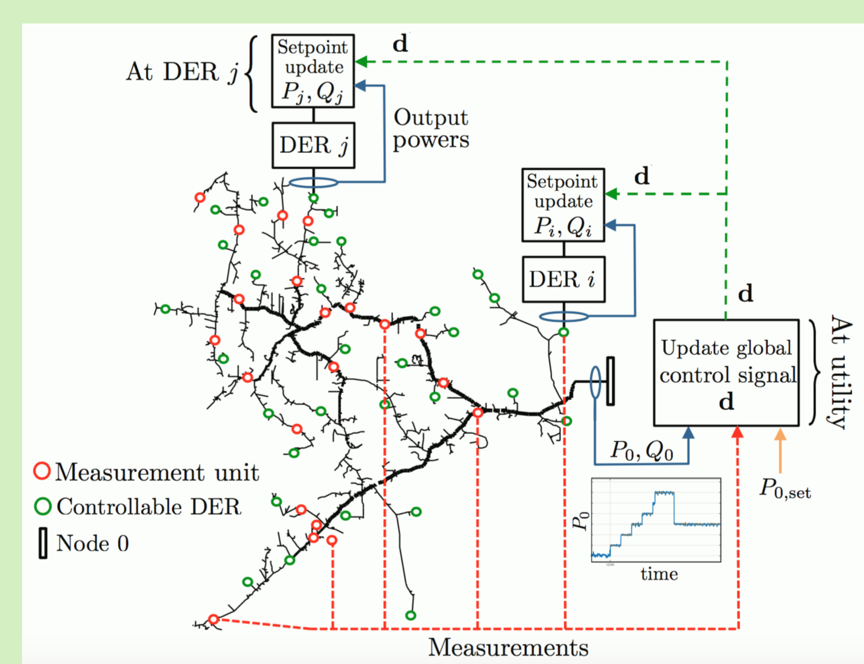
Utilize *feedback measurements*: replace $\mathcal{F}(\hat{x}_{t-1}; t)$ by measured values $\check{y}_t$

Feedback-based primal-dual gradient method:

$$\hat{x}_t = \mathcal{P}_{\mathcal{X}_t}\left[\hat{x}_{t-1} - \alpha\left(\nabla c_t(\hat{x}_{t-1}) + (H_t(\check{y}_t) J_t(\hat{x}_{t-1}, \check{y}_t))^T \hat{\lambda}_{t-1}\right)\right]$$
$$\hat{\lambda}_t = \mathcal{P}_{\mathbb{R}^m_+}\left[\hat{\lambda}_{t-1} + \beta\left(h_t(\check{y}_t) - \epsilon\hat{\lambda}_{t-1}\right)\right]$$

- H_t: Jacobian matrix of h_t
- J_t: Jacobian matrix of $\mathcal{F}(\,\cdot\,; t)$

- Fast timescale measurements monitor system's *time-varying* behavior.
- Measurements are fed back to optimization iterations *in real-time* even if iteration has not converged yet.
- The resulting $\hat{x}_t$ ***tracks*** time-varying optimal solution.
- Feedback measurements help reduce computation time and facilitate distributed implementation.



A distribution network control system [2] based on the proposed algorithm.

Theoretical Analysis

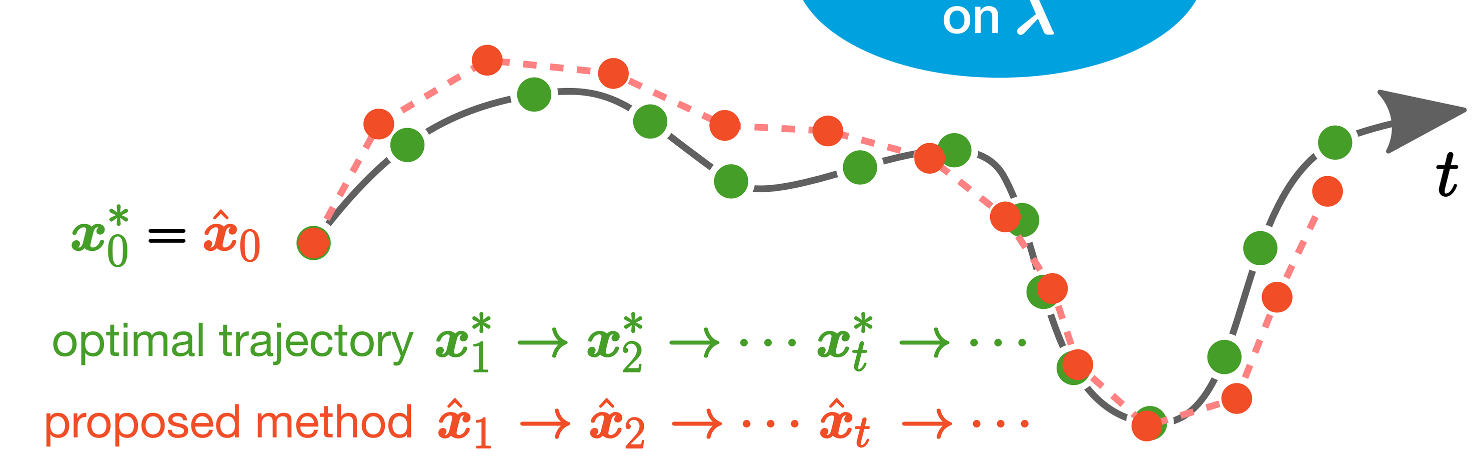
Theorem: Assume certain regularity conditions and that $\mathcal{L}_t$ is locally sufficiently convex in x around (x_t^*, λ_t^*) for any t. With properly chosen parameters α, β, ϵ, if

$$\sigma := \sup_t \|x_t^* - x_{t-1}^*\|$$

is less than some threshold, and measurement error is upper bounded by Δ, then the tracking error satisfies

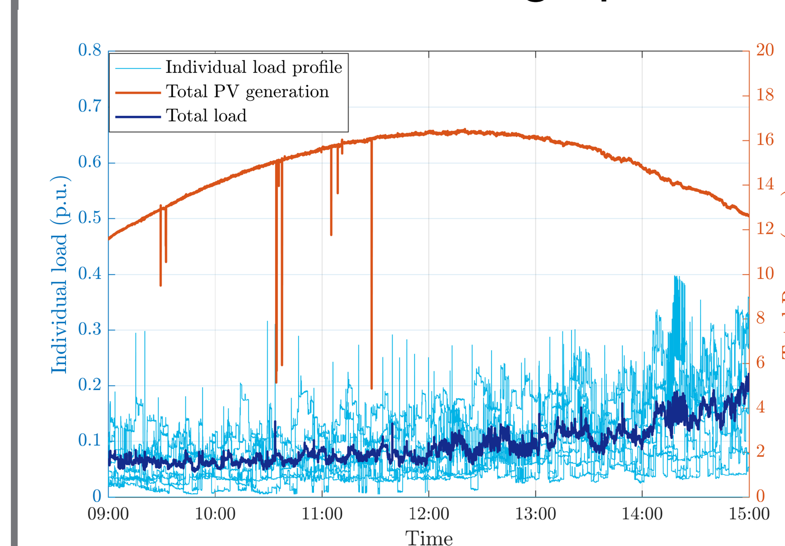
$$\|\hat{x}_t - x_t^*\| \le C_1\left(\sigma + C_2 \sup_t \|\lambda_t^*\| + C_3\Delta\right)$$

tracking error; how fast the optimal solution moves; error due to regularization on λ; error due to measurement noise

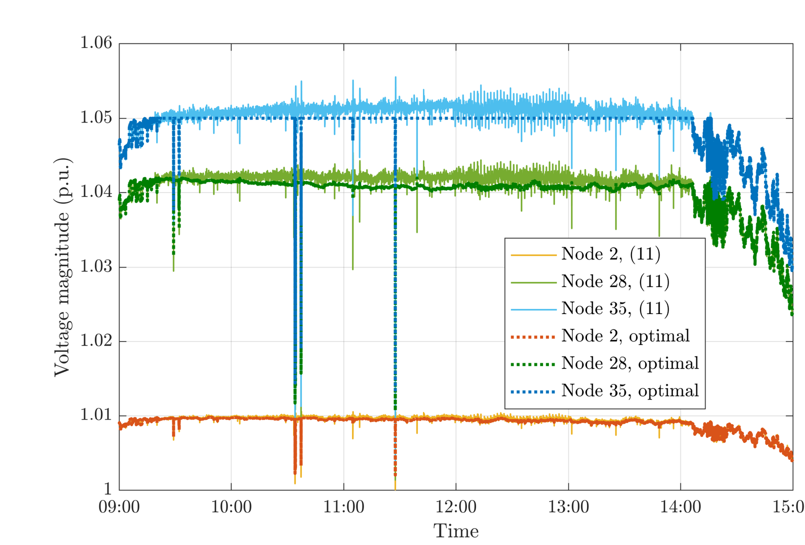


Simulation

The proposed algorithm has been simulated on a real-time optimal power flow (OPF) problem on a modified IEEE 37 node test feeder with high penetration of solar panels.



solar generation and load profiles



voltage profiles of the proposed method

[1] Y. Tang, E. Dall'anese, A. Bernstein, and S. Low. "A feedback-based regularized primal-dual gradient method for time-varying nonconvex optimization", submitted to CDC 2018.
[2] E. Dall'Anese, S. Guggilam, A. Simonetto, Y. C. Chen, and S. V. Dhople. "Optimal regulation of virtual power plants", IEEE Transactions on Power Systems.